Solving Inequalities
Physical Fitness Zones

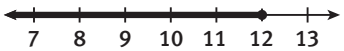
ACTIVITY 1.6

SUGGESTED LEARNING STRATEGIES: Shared Reading, Questioning the Text, Think/Pair/Share, Group Presentation

My Notes

Spartan Middle School students participate in Physical Education testing each semester. In order to pass, 12- and 13-year-old girls have to do at least 7 push-ups and 4 modified pull-ups. They also have to run one mile in 12 minutes or less.

You can use an inequality to express the passing marks in each test.

	Push-Up, p	Modified Pull-Up, m	One-Mile Run, r
verbal	at least 7 push-ups	at least 4 pull-ups	12 minutes or less
inequality	$p \geq 7$	$m \geq 4$	$r \leq 12$
Graph			

1. Why do you think the graphs of push-ups and pull-ups are dotted but the graph of the mile run is a solid ray?

MATH TERMS

The **graph of an inequality** in one variable is all the points on a number line that make the inequality true.

2. Jamie ran one mile in 12 minutes 15 seconds, did 8 push-ups and 4 modified pull-ups. Did she pass the test? Explain.

The **solution of an inequality** in one variable is the set of numbers that make the inequality true.

MATH TIP

To verify a solution of an inequality, substitute the value into the inequality and simplify to see if the result is a true statement.

3. Use the table below to figure out which x-values are solutions to the equation and which ones are solutions to the inequality. Show your work in the rows of the table.

x-values	Solution to the equation? $2x + 3 = 5$	Solution to the inequality? $2x + 3 > 5$
1		
2		
-1		
0		
8.5		

My Notes

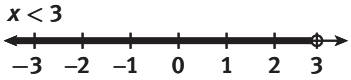
SUGGESTED LEARNING STRATEGIES: Quickwrite, Think/Pair/Share

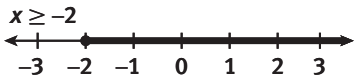
4. How many solutions are there to the equation $2x + 3 = 5$? Explain.

5. Are 2 and 8.5 the only solutions to the inequality $2x + 3 > 5$? Explain.

6. Would 1 be a solution to the inequality $2x + 3 \geq 5$? Explain.

Here are the number line graphs of two different inequalities.





7. Compare and contrast the two inequalities and graphs that are shown above.

Similarities	Differences

SUGGESTED LEARNING STRATEGIES: Quickwrite, Create Representations, Guess and Check

My Notes

8. Think about why the graphs are different.

a. Why is one of the graphs showing a solid ray going to the left and the other graph showing a solid ray going to the right?

b. Why does one graph have an open circle and the other graph a filled-in circle?

TRY THESE **A**

Graph each inequality on a number line.

a. $x < -2$ **b.** $x \geq 5$

WRITING MATH

An open circle represents $<$ or $>$ inequalities, and a solid circle represents $\leq$ or $\geq$ inequalities.

9. Chloe and Charlie are taking a trip to the pet store to buy some things for their new puppy. They know that they need a bag of food that costs \$7, and they also want to buy some new toys for the puppy. They find a bargain barrel containing toys that cost \$2 each.

a. Write an expression for the amount of money they will spend if the number of toys they buy is t.

b. Chloe has \$30 with her and Charlie has one-third of this amount with him. Use this information and the expression you wrote in part (a) to write an inequality for finding the number of toys they can buy.

There are different methods for solving the inequality you wrote in the previous question. Chloe suggested that they guess and check to find the number of new toys that they could buy.

10. Use Chloe's suggestion to find the number of new puppy toys that Chloe and Charlie can buy with their combined money.

My Notes

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Look for a Pattern, Think/Pair/Share

Charlie remembered that they could use algebra to solve inequalities. He imagined that the inequality symbol was an equal sign. Then he used equation-solving steps to solve the inequality.

11. Use Charlie's method to solve the inequality you wrote in Item 9b.

12. Did you get the same answer using Charlie's method as you did using Chloe's method? Explain.

TRY THESE **B**

Solve and graph. Remember to substitute some sample answers back into the original inequality to check your work.

a. $3 + 4x < 7$ **b.** $2(x - 3) + x \geq 6$ **c.** $5 > 2 + \frac{2}{3}x$

Chloe liked the fact that Charlie's method for solving inequalities did not involve guess and check, so she asked him to show her the method. She suggested that they solve the following inequality:

Charlie showed Chloe the work below for $-2x - 4 > 8$:

$$\begin{aligned} -2x - 4 &> 8 \\ -2x - 4 + 4 &> 8 + 4 \\ -2x &> 12 \\ \frac{-2x}{-2} &> \frac{12}{-2} \\ x &> -6 \end{aligned}$$

When Chloe went back to check the solution by substituting a value for x back into the original inequality, she found that something was wrong.

13. Confirm or disprove Chloe's conclusion by substituting values for x into the original inequality.

SUGGESTED LEARNING STRATEGIES: Quickwrite, Identify a Subtask

My Notes

Chloe tried the problem again but used a few different steps.

$$\begin{aligned} -2x - 4 &> 8 \\ -2x + 2x - 4 &> 8 + 2x \\ -4 &> 8 + 2x \\ -4 - 8 &> 8 - 8 + 2x \\ -12 &> 2x \\ \frac{-12}{2} &> \frac{2x}{2} \\ -6 &> x \end{aligned}$$

Chloe concluded that $x < -6$

14. Is Chloe's conclusion correct? Explain.

15. Explain what Chloe did to solve the inequality.

Charlie looked back at his work. He said that he could easily fix his work by simply switching the inequality sign.

16. What do you think about Charlie's plan? Explain.

Although all of these methods worked, Charlie and Chloe wanted to know why they were working.

My Notes

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Group Presentation

Here is an experiment to discover what went wrong with Charlie's first method. Look at what happens when you multiply or divide by a negative number.

Directions	Numbers	Inequality
Pick two different numbers.	2 and 4	$2 < 4$
Multiply both numbers by 3.	2(3) and 4(3)	$6 < 12$
Multiply both numbers by -3.	$2(-3)$ and $4(-3)$	$-6 > -12$

17. Try this experiment again with two different numbers. Record your results below. Compare your results to the rest of your class.

18. What happens when you multiply by a negative number? What happens when you divide by a negative number?

19. How does this affect how you solve an inequality?

EXAMPLE 1

Solve and graph: $-3x + 5 \leq 20$

Step 1: *Subtract 5 from both sides.*

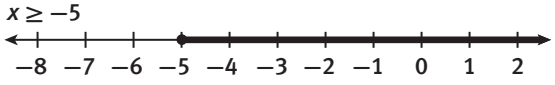
$$-3x + 5 - 5 \leq 20 - 5$$
$$-3x \leq 15$$

Step 2: *Divide both sides by -3.*

Remember to reverse the inequality sign.

$$\frac{-3x}{-3} \geq \frac{15}{-3}$$
$$x \geq -5$$

Solution: $x \geq -5$



TRY THESE C

Solve and graph.

a. $3 - 4x \leq 11$ **b.** $6 - 3(x + 2) > 15$ **c.** $2(x + 5) < 8(x - 3)$

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Interactive Word Wall, Look for a Pattern

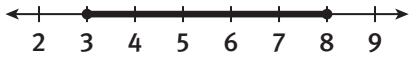
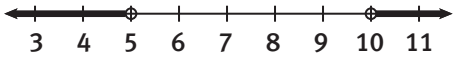
My Notes

ACADEMIC VOCABULARY

compound inequality

Compound inequalities are two inequalities joined by the word *and* or by the word *or*. Inequalities joined by the word *and* are called **conjunctions**. Inequalities joined by the word *or* are **disjunctions**. You can represent compound inequalities using words, symbols or graphs.

20. Complete the table. The first two rows have been done for you.

Verbal Description	Some Possible Solutions	Inequality	Graph
all numbers from 3 to 8, inclusive	3.5, 4, $4\frac{1}{3}$, 5, 6, 7.9, 8	$x \geq 3$ and $x \leq 8$	
all numbers less than 5 or greater than 10	−2, 0, 3, 4, 4.8, $10\frac{3}{4}$, 11	$x < 5$ or $x > 10$	
all numbers greater than −1 and less than or equal to 4			
all numbers less than or equal to 3 or greater than 6			

21. Compare and contrast the graphs for conjunctions and disjunctions.

Similarities	Differences

My Notes

SUGGESTED LEARNING STRATEGIES: Questioning the Text, Activating Prior Knowledge, Create Representations, Group Presentation

EXAMPLE 2

Spartan Middle School distributes this chart to students each year.

Age	Mile Run (min:sec)		Push-Ups		Modified Pull-Ups	
	Boys	Girls	Boys	Girls	Boys	Girls
12	8:00–10:30	9:00–12:00	10–20	7–15	7–20	4–13
13	7:30–10:00	9:00–12:00	12–25	7–15	8–22	4–13

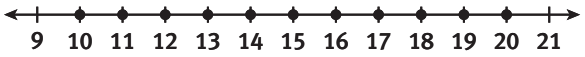
Write and graph a compound inequality that describes the push-up range for 12-year-old boys.

Step 1: *Choose a variable.*

Let p represent the number of push-ups for 12 year old boys.

Step 2: *Determine the range and write an inequality.*

The push-up range is $10 \leq p \leq 20$.



Solution: The compound inequality is $10 \leq p \leq 20$. The graph is shown above.

WRITING MATH

The compound inequality "$p \geq 10$ and $p \leq 20$" can also be written as "$10 \leq p \leq 20$" because the two inequalities $p \geq 10$ and $10 \leq p$ are equivalent.

CONNECT TO AP

In upper-level mathematics classes, inequalities are expressed in interval notation. The interval notation for $x > 3$ is $(3,\infty)$.

TRY THESE **D**

Write and graph a compound inequality for each range or score.

a. the push-up range for 13 year old boys

b. the pull-up range for 13 year old girls

c. the mile run range for 12 year old girls

d. the mile run range for 13 year old boy

e. a score outside the healthy fitness zone for girl's push-ups

SUGGESTED LEARNING STRATEGIES: Note Taking, Create Representations, Think/Pair/Share

My Notes

22. Why are individual dots used in the graphs for Example 2 and some of the graphs in Try These D?

To solve a conjunction, break the compound inequality into two parts and solve each part. The solution of the conjunction will be the solutions that are *common to both parts*.

EXAMPLE 3

Solve and graph the conjunction: $3 < 3x - 6 < 8$

Step 1: *Break the compound inequality into two parts.*

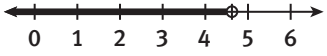
$3 < 3x - 6$ and $3x - 6 < 8$

Step 2: *Solve and graph* $3 < 3x - 6$.

$$3 < 3x - 6$$
$$3 + 6 < 3x - 6 + 6$$
$$9 < 3x$$
$$3 < x \text{ or } x > 3$$

Step 3: *Solve and graph* $3x - 6 < 8$.

$$3x - 6 < 8$$
$$3x - 6 + 6 < 8 + 6$$
$$3x < 14$$
$$x < 4\frac{2}{3}$$



Step 4: *Determine what is common to the solutions of each part.*

What is common in Steps 2 and 3?

Solution: $3 < x < 4\frac{2}{3}$

TRY THESE E

Solve and graph each conjunction.

a. $-1 < 3x + 5 < 6$

b. $2 < \frac{x}{3} - 5 < 6$

To solve a disjunction, solve and graph each part. The solution of the disjunction will be *all the solutions from both parts*.

My Notes

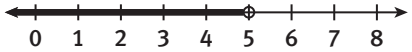
SUGGESTED LEARNING STRATEGIES: Note Taking, Create Representations, Think/Pair/Share, Group Presentation

EXAMPLE 4

Solve and graph the compound inequality: $2x - 3 < 7$ or $4x - 4 \geq 20$.

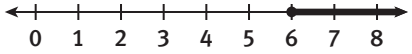
Step 1: *Solve and graph* $2x - 3 < 7$.

$$2x - 3 + 3 < 7 + 3$$
$$2x < 10$$
$$x < 5$$

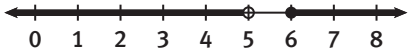


Step 2: *Solve and graph* $4x - 4 \geq 20$.

$$4x - 4 \geq 20$$
$$4x - 4 + 4 \geq 20 + 4$$
$$4x \geq 24$$
$$x \geq 6$$



Step 3: *Combine the solutions.*



Solution: $x < 5$ or $x \geq 6$

TRY THESE **F**

Solve and graph each compound inequality.

a. $5x + 1 > 11$ or $x - 1 < -4$ **b.** $-5x > 20$ or $x + 12 \geq 7$

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

Solve and graph each inequality on a number line.

1. $3x - 8 + 4x > 6$

2. $5 < 3x + 8$

3. $4 - 2(x + 1) < 18$

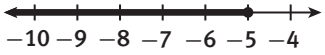
4. $\frac{2}{3} \geq \frac{1}{6} - 2x$

5. $-2 < 2(x + 5) \leq 7$

6. $5x \geq 14 - 2x$ or $3(x + 3) \leq 6$

7. Correct the mistakes in the problem.

$$2x + 5 - 6x > 25$$
$$-4x + 5 > 25$$
$$-4x > 20$$
$$x > -5$$



8. Explain why you reverse the inequality when you multiply or divide both sides of an inequality by a negative number.

9. What is the largest number in the solution set of $x < 3$?

10. MATHEMATICAL REFLECTION Describe the differences between solving and graphing a conjunction and a disjunction.